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Intuitionistic Fuzzy Metric Space and Absorbing Maps

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Abstract—*This paper deals with common fixed point of six mappings by using absorbing maps in an intuitionistic fuzzy metric space.*

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1. INTRODUCTION AND PRELIMINARIES

In 1965, Zadeh[16] introduced the concept of fuzzy sets as a new way to represent vagueness in our everyday life. Since then, many authors regarding the theory of fuzzy sets and its applications have developed a lot of literature. Kramosil and Michalek[10] , Erceg[5], Deng[4], Kaleva and Seikkla[11] have introduced fuzzy metric space in different ways. Atanassov [2]initiated and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets In 2004, Park [13] introduced a notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [10].In 2008, Mishra, Ranadive and Gopal[18] introduced the notion of absorbing maps. They explore the possibility of applying the notion of reciprocal continuity and absorbing maps for finding common fixed points of four mappings.

Here, we remind some basic definitions and well known results in intuitionist fuzzy metric space.

Definition 1.1. [16] Let X be any non empty set. A fuzzy set A in X is a function with domain X and values in [0,1].

Definition 1.2. [2] Let X be any set. An intuitionistic fuzzy set A of X is an object of the form

A = $\left\{\left(x,\mu_A(x),\gamma_A(x)\right)\middle| x\in X\right\}$, where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership and the non-membership of the element x $\in$E respectively and for every x $\in$X, $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

Definition 1.3. [13] A triangular norm * (shortly t-norm) is a binary operation on the unit interval [0, 1] such that for all a, b, c, d $\in$ [0, 1] the following conditions are satisfied:

(1) a ∗ 1 = a ;

(2) a ∗ b = b ∗ a ;

(3) a ∗ b ≤ c ∗ d whenever a ≤ c and b ≤ d ;

(4) a ∗ (b ∗ c) = (a ∗ b) ∗ c .

Definition1.4.[13] A binary operation ◊ : [0, 1] × [0, 1] → [0, 1] is a continuous t-conorm if it satisfies the following conditions:

(1) ◊ is associative and commutative,

(2) ◊ is continuous,

(3) a ◊ 0 = a for all a $\in$ [0, 1],

(4) a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d, for each a, b, c, d $\in$ [0, 1].

Example 1.5[1] Two typical examples of continuous t-conorm are a◊ b = min(a+b, 1)and a ◊ b = max(a, b).

Definition 1.6 [1] A 5-tuple (X,M,N, *, ◊) is called a intuitionistic fuzzy metric space if X is an arbitrary (non-empty) set, * is a continuous t-norm, ◊ a continuous t-conorm and M, N are fuzzy sets on $X^2\times(0,\infty)$, satisfying the following conditions:

for each x, y, z $\in$ X and t, s > 0,

(a)M(x, y, t) + N(x, y, t) ≤ 1,

(b) M(x, y, t) > 0,

(c) M(x, y, t) = 1 if and only if x = y,

(d) M(x, y, t) = M(y, x, t),

(e) M(x, y, t) *M(y, z, s) ≤ M(x, z, t + s),

(f) M(x, y, .) : (0,∞) → [0, 1] is continuous.

(g) N(x, y, t) > 0,

(h) N(x, y, t) = 0 if and only if x = y,

(i) N(x, y, t) = N(y, x, t),

(j) N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t + s),

(k) N(x, y, .) : (0,∞) → [0, 1] is continuous.

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Definition 1.7 [1] Let (X, M,N, $*$,◊) be an intuitionistic fuzzy metric space:

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$), if $\lim_{n\to\infty} M(x_n, x, t) = 1$ $\lim_{n\to\infty} N(x_n, x, t)$=0, for all t > 0.

(ii) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n\to\infty} N(x_{n+p}, x_n, t)$ =0 for all t > 0, p>0.

(iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete .

Definition 1.8. A pair (A, S) of self maps of an intuitionistic fuzzy metric space (X, M, N, $*$,◊) is said to be reciprocal continuous if

$\lim_{n\to\infty} ASx_n$=Ax and $\lim_{n\to\infty} SAx_n$=Sx, whenever there exists a sequence $\{x_n\} \in$ X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$=x for some x ∈ X.

If A and S are both continuous then they are obviously reciprocally continuous but the converse need not be true.

Example 1.9 Let X=[4,30] and d be the usual metric space X. Define mappings A,S : X → X by

Ax=x if x=4 Sx=x if x=4

Ax=5 if x>4 Sx=10 if x>4

It may be noted that A and S are reciprocally continuous mappings but neither A nor S is continuous mappings.

Definition 1.10. Let f and g are two self maps on an intuitionistic fuzzy metric space (X, M, N, $*$,◊) then f is called g-absorbing if there exists a positive integer R > 0 such that

M(gx, gfx, t) ≥ M(gx, fx, t/R) and

N(gx, gfx, t) ≤ N(gx, fx, t/R for all x ∈ X.

Similarly, we can defined f-absorbing maps.

The map f is called point wise g-absorbing if for x∈X, there exists a positive integer R > 0 such that

M(gx, gfx, t) ≥M(gx, fx, t/R) and

N(gx, gfx, t) ≤N(gx, fx, t/R) for all x ∈ X,

Similarly, we can defined point wise f-absorbing maps.

Lemma 1.11 [13] If for all x, y∈X , t>0 and 0< k< 1, M(x, y, kt)≥M(x, y, t),N(x, y, kt) ≤ N(x, y, t),then x =y.

Lemma 1.12 [13] : M(x, y, .) is non-decreasing and N(x, y, .) is non-increasing for all x, y in X.

2. MAIN RESULTS

Theorem 2.1. Let P be point wise AB - absorbing and Q be point wise ST–absorbing self maps on a complete intuitionistic fuzzy metric space (X, M,N, *, ◊) with continuous t-norm and t-conorm are defined by a$*$b = min{a,b}, a◊b = max{a,b} where a,b ∈ [0,1] satisfying the conditions:

(2.1) P(X)⊆ ST(X), Q(X) ⊆ AB(X);

(2.2) There exists k ∈ (0,1) such that for every x, y∈X and t > 0,

M(Px,Qy,kt)≥min{M(ABx,STy,t), M(Px,ABx,t), M(Qy, STy, t),M(Px, STy, t)},

N(Px,Qy,kt)≤max{N(ABx, STy, t),N(Px, ABx, t),N(Qy, STy, t), N(Px, STy, t)};

(2.3) For all x,y∈ X, $\lim_{t\to\infty}$ M(x, y, t) = 1,

$\lim_{t\to\infty}$ N(x, y, t) = 0;

(2.4) AB=BA, ST=TS, PB=BP, SQ=QS, QT=TQ.

If the pair of maps (P, AB) is reciprocal continuous compatible maps then P, Q, S, T, A and B have a unique common fixed point in X. Proof: let x_0 be any arbitrary point in X, construct a sequence $y_n \in X$ such that $y_{2n-1} = STx_{2n-1} = Px_{2n-2} and y_{2n} = ABx_{2n} = Qx_{2n+1}$, n = 1, 2, 3. This can be done by the virtue of (2.1). By using contractive condition we obtain,

M(y_{2n+1},y_{2n+2},kt) = M(P x_{2n}, Qx_{2n+1}, kt)

≥min{M(AB x_{2n}, ST x_{2n+1}, t),M(P x_{2n}, AB x_{2n}, t), M(Q $x_{2n+1,} STx_{2n+1}$,t), M(P x_{2n} , ST x_{2n+1} , t)}

≥ min {M(y_{2n}, y_{2n+1}, t),M(y_{2n+1}, y_{2n}, t),

M(y_{2n}, y_{2n+1} ,t), 1}

which implies,

M(y_{2n+1}, y_{2n+2}, kt) $\geq$ M(y_{2n}, y_{2n+1}, t)

in general,

M(y_n, y_{n+1}, kt) $\geq$ M(y_{n-1}, y_n, t)

and

N(y $_{2n+1}$,y $_{2n+2}$,kt) = N(P x_{2n}, Qx_{2n+1}, kt)

$\leq$max {N(AB x_{2n} ,ST x_{2n+1}, t),N(P x_{2n} ,AB x_{2n}, t), N(Q $x_{2n+1,} STx_{2n+1}$,t), N(P x_{2n} , ST x_{2n+1} , t)}

$\leq$max{N(y_{2n}, y_{2n+1},t),N(y_{2n+1}, y_{2n},t),

N(y_{2n}, y_{2n+1},t), 0}

which implies,

N(y_{2n+1}, y_{2n+2}, kt) $\leq$ N(y_{2n}, y_{2n+1}, t)

in general,

N(y_n, y_{n+1}, kt) $\leq$ N(y_{n-1}, y_n, t) ....(1)

To prove {y_n} is a Cauchy sequence, we have to show

M(y_n, y_{n+1}, t)$\rightarrow$1 and N(y_n, y_{n+1}, t)$\rightarrow$0(for t > 0 as n $\rightarrow$ ∞ uniformly on p$\in$N), for this from (1) we have,

M(y_n, y_{n+1},t)$\geq$M (y_{n-1}, y_n, t/k) $\geq$M(y_{n-2}, y_{n-1}, t/k^2)$\geq$ …….$\geq$ M(y_0, $y_1, \frac{t}{k^n}$)$\rightarrow$1as n$\rightarrow\infty$ for p $\in$N, by (1) we have

M(y_n, y_{n+p},t)$\geq$M(y_n, y_{n+1},(1-k)t)$*$ M(y_{n+1}, y_{n+p},kt) $\geq$ M($y_{0,} y_1, \frac{(1-k)t}{k^n}$)$*$ M(y_{n+1}, y_{n+2},t)$*$

M(y_{n+2}, y_{n+p}, (k-1)t)

$\geq$M($y_{0,} y_1, \frac{(1-k)t}{k^n}$)$*$ M($y_0, y_{1,} \frac{t}{k^n}$)$*$ M(y_{n+2}, y_{n+3},t) $* M(y_{n+3}, y_{n+p}, (k-2)t)$ $\geq$M($y_{0,} y_1, \frac{(1-k)t}{k^n}$)$*$ M($y_0, y_{1,} \frac{t}{k^n}$)$*$ M($y_0, y_1, \frac{(1-k)t}{k^{n+2}}$)……$*$ $M(y_0, y_1, \frac{(k-p)t}{k^{n+p+1}}$).

Thus M (y_n, y_{n+p}, t) $\rightarrow$ 1 (for all t > 0 as n$\rightarrow$ ∞ uniformly on p $\in$ N)

and

N(y_n, y_{n+1}, t) $\leq$N (y_{n-1}, y_n, t/k) $\leq$N(y_{n-2}, y_{n-1}, t/k^2) $\leq$ ……. $\leq$ N(y_0, $y_1, \frac{t}{k^n}$)$\rightarrow$0as n$\rightarrow\infty$ for p $\in$N, by (1) we have

N(y_n, y_{n+p}, t) $\leq$N(y_n, y_{n+1},(1-k)t) ◊ N(y_{n+1}, y_{n+p}, kt)$\leq$ N($y_{0,} y_1, \frac{(1-k)t}{k^n}$)◊N($y_{n+1}, y_{n+2}$,t) ◊

N(y_{n+2}, y_{n+p}, (k-1)t)

$\leq$N($y_{0,} y_1, \frac{(1-k)t}{k^n}$)◊N($y_0, y_{1,} \frac{t}{k^n}$)◊N($y_{n+2}, y_{n+3}$,t) ◊ $N(y_{n+3}, y_{n+p}, (k-2)t)$

$\leq$N($y_{0,} y_1, \frac{(1-k)t}{k^n}$)◊N($y_0, y_{1,} \frac{t}{k^n}$)◊

N($y_0, y_1, \frac{(1-k)t}{k^{n+2}}$)……◊ $N(y_0, y_1, \frac{(k-p)t}{k^{n+p+1}}$)

Thus N(y_n, y_{n+p}, t) $\rightarrow$ 0 (for all t > 0 as n$\rightarrow$ ∞ uniformly on p $\in$ N). Therefore, {y_n} is a Cauchy sequence in X. But (X, M,N, *,◊) is complete so there exists a point (say) z in X such that {y_n}$\rightarrow$z. Also we have $\{Px_{2n-2}\}, \{STx_{2n-1}\}, \{ABx_{2n}\}, \{Qx_{2n+1}\} \rightarrow z$. Since the pair (P, AB) is reciprocally continuous mappings, then we have, $\lim_{n\rightarrow\infty} PABx_{2n} = Pz$ and $\lim_{n\rightarrow\infty} ABPx_{2n} = ABz$ and compatibility of P and AB yields,

$\lim_{n\rightarrow\infty} M(PABx_{2n}, ABPx_{2n}, t) = 1$ and

$\lim_{n\rightarrow\infty} N(PABx_{2n}, ABPx_{2n}, t) = 0$

i.e. M(Pz, ABz, t) = 1 and N(Pz, ABz, t) = 0. Hence Pz = ABz. Since P(X) $\subseteq$S T(X) then there exists a point u in X such that Pz =STu.

Now by contractive condition, we get

M(Pz, Qu, kt) $\geq$ min {M(ABz, STu, t), M(Pz, ABz, t), M (Qu,STu, t), M(Pz, STu, t)}

$\geq$ min {M(Pz, Pz, t), M(Pz, Pz, t),M(Qu,Pz, t), M(Pz, Pz, t) }> M(Pz, Qu, t)

and

N(Pz, Qu, kt)$\leq$ max {N(ABz, STu, t), N(Pz, ABz, t),N(Qu,STu, t), N(Pz, STu, t)}

$\leq$ max {N(Pz, Pz, t), N(Pz, Pz, t),N(Qu,Pz, t), N(Pz, Pz, t)} < N(Pz, Qu, t)

i.e. Pz = Qu. Thus Pz = ABz = Qu =STu.

Since P is AB - absorbing then for R > 0, we have

M(ABz, ABPz, t) $\geq$ M(ABz, Pz, t/R) = 1 and N(ABz, ABPz, t) $\leq$ N(ABz, Pz, t/R) =0

i.e. Pz = ABPz = ABz.

Now by contractive condition, we have,

M(PPz, Pz, t) = M(PPz, Qu, t)$\geq$ min {M(ABPz, STu, t), M(PPz, ABPz, t),M(Qu,STu, t), M(PPz, STu, t)}

$\geq$min{M(Pz,Pz,t),(PPz,Pz,t),M(Qu,Qu,t),M(PPz,Pz,t)}

= M (PPz, Pz, t)

and

N(PPz, Pz, t) = N(PPz, Qu, t)

$\leq$ max {N(ABPz, STu, t), N(PPz, ABPz, t),N(Qu,STu, t), N(PPz, STu, t)}

$\leq$max{N(Pz,Pz,t),N(PPz,Pz,t),N(Qu,Qu,t),N(PPz,Pz,t)} = N(PPz, Pz, t),

i.e. PPz=Pz=ABPz.

Therefore Pz is a common fixed point of P and AB. Similarly, ST is Q–absorbing, therefore, we have,

M(STu, STQu, t) $\geq$M(STu, Qu, t/R) = 1

i.e. STu = STQu = Qu.

Now by contractive condition, we have

M(Qu, QQu, t) = M(Pz, QQu, t)

$\geq$min{M(ABz, STQu, t),M(Pz, ABz, t), M(QQu,STQu, t), M(Pz, STQu, t)}

=min{M(ABz, Qu, t),M(Pz, Pz, t),M(QQu, Qu, t), M(Pz, Qu, t)}

=min{M(Pz,Pz,t),M(Pz,Pz,t),M(QQu,Qu,t),M(Pz, Pz, t)}

= min{1,1,M(QQu, Qu, t),1} = M(QQu, Qu, t)

and

N(STu, STQu, t) $\leq$N(STu, Qu, t/R) =0

i.e. STu = STQu = Qu.

Now by contractive condition, we have

N(Qu, QQu, t) = N(Pz, QQu, t)

$\leq$max{N(ABz,STQu,t),N(Pz,ABz,t),N(QQu,STQu,t), N(Pz, STQu, t)}

= max{N(ABz, Qu, t),N(Pz, Pz, t),N(QQu, Qu, t),N(Pz, Qu, t)}

= max{N(Pz, Pz,t),N(Pz,Pz,t),N(QQu,Qu,t), N(Pz, Pz, t)}

=max{0,0,N(QQu, Qu, t),0}= N(QQu, Qu, t).

i.e. QQu = Qu = STQu .

Now putting Pz=Qu ,we have

QPz=Pz=STPz.

Now putting x=BPz, y = Pz in (2.2), we have

M(P(BPz),Q(Pz),kt)$\geq$ min{M(AB(BPz),ST(Pz),t),M(P(BPz),AB(BPz),t),M(Q(Pz),ST(Pz),t),M(P(BPz),ST(Pz),t)}

As PBPz = BPPz = BPz and ABBPz = BABPz = Pz, we have

M(BPz,Pz,kt)$\geq$min{M(Pz,Pz,t),M(BPz,Pz,t), M(Pz,Pz,t), M(BPz,Pz,t)}

$\geq$ min {1, M(BPz,Pz,t),1,M(BPz,Pz,t)}

$\geq$ M(BPz,Pz,t)

and

N(P(BPz),Q(Pz),kt)

$\leq$max{N(AB(BPz),ST(Pz),t),N(P(BPz),AB(BPz),t), N(Q(Pz),ST(Pz),t), N(P(BPz),ST(Pz),t)}

As PBPz = BPPz = BPz and ABBPz = BABPz = Pz, we have

N(BPz,Pz,kt)$\leq$max{N(Pz,Pz,t),N(BPz,Pz,t),N(Pz,Pz,t), N(BPz,Pz,t)}

$\leq$ max {0, N(BPz,Pz,t),0,N(BPz,Pz,t)}

$\leq$N(BPz,Pz,t).

By using lemma 1.12, we have

i.e. BPz=Pz.

Since Pz = PPz = QPz =ABPz =APz

hence, Pz = PPz = QPz =BPz =APz

Now putting x = Pz, y=TPz in (2.2), we have M(PPz,QTPz,kt)

$\geq$min{M(ABPz,STTPz,t),M(PPz,ABPz,t), M(QTPz,STTPz,t),M(PPz,STTPz,t)}

As STTPz = TSTPz = TPz and QTPz = TQPz = TPz , we have

M(Pz,TPz,kt)$\geq$min{M(Pz,TPz,t),M(Pz,Pz,t), M(TPz,TPz,t), M(Pz,TPz,t)}

M(Pz,TPz,kt) $\geq$ min {M(Pz,TPz,t), 1,1, M(Pz,TPz,t)}

$\geq$ M(Pz,TPz,t)

and

N(PPz,QTPz,kt)

$\leq$max{N(ABPz,STTPz,t),N(PPz,ABPz,t), N(QTPz,STTPz,t), N(PPz,STTPz,t)}

As STTPz = TSTPz = TPz and QTPz = TQPz = TPz , we have

N(Pz,TPz,kt)$\leq$max{N(Pz,TPz,t), N(Pz,Pz,t),N(TPz,TPz,t),N(Pz,TPz,t)}

N(Pz,TPz,kt) $\leq$ max {N(Pz,TPz,t), 0, 0, N(Pz,TPz,t)}

$\leq$ N(Pz,TPz,t)

By using lemma 1.12, we have

i.e. TPz=Pz.

Since Pz = PPz = QPz =BPz =APz = TPz = STPz hence Pz = PPz = QPz =BPz =APz = TPz = SPz. Hence Pz is a common fixed point of P, Q, S ,T ,A and B. Uniqueness of Pz can easily follows from contractive condition. The proof is similar when Q and ST are assumed compatible and reciprocally continuous. This completes the proof.

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